Practice Set 16 Small Sample Hypothesis Testing Using Student's t Test

I. Darin wants to determine whether there is a difference in the number of sick days taken by employees based upon their education. A sample of 11 high school graduates had a mean of 5 sick days per year and a standard deviation of 2.5 days. Twelve non-graduates averaged 10 sick days per year. Their standard deviation was 3.25 days. Is there a difference in sick days taken based upon education? Use the .01 level of significance.

Given
$n_1 = 11$
$\bar{X}_1 = 5$
$S_1^2 = 2.5^2 = 6.25$
$n_2 = 12$
$\bar{X}_2 = 10$
$S_2^2 = 3.25^2 = 10.56$
$\alpha = .01$

$$s_w^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2} = \frac{(11-1)6.25+(12-1)10.56}{11+12-2} = \frac{62.50+116.16}{21} = 8.51$$

The 5-step approach to hypothesis testing

1. $H_o: \mu_1 = \mu_2$ and $H_1: \mu_1 \neq \mu_2$
2. $\alpha = .01$
3. The test statistic is $\bar{x}$.
4. $df = n_1 + n_2 - 2 = 11 + 12 - 2 = 21$
   $\alpha = .01$ and $.01 \div 2 = .005 \rightarrow t = \pm 2.831$
5. Apply the decision rule.

$$t = \frac{\bar{X}_1-\bar{X}_2}{\sqrt{s_w^2(\frac{1}{n_1}+\frac{1}{n_2})}} = \frac{5-10}{\sqrt{8.51(\frac{1}{11}+\frac{1}{12})}} = -4.11$$

Reject H_0 because -4.11 is beyond - 2.831. Non-high school graduates took a different number of sick days than high school graduates.

II. Darin conducted a training program for 5 recently-hired employees. Test at the .01 level whether the training program increased employee efficiency.

Employee	Efficiency Rating Before	After	d	d^2
1	8	9	-1	1
2	6	8	-2	4
3	7	8	-1	1
4	7	9	-2	4
5	8	10	-2	4
			-8	14

$$\bar{d} = \frac{\sum d}{n} = \frac{-8}{5} = -1.6$$

$$s_d = \sqrt{\frac{\sum d^2 - \frac{(\sum d)^2}{n}}{n-1}} = \sqrt{\frac{14-\frac{-8^2}{5}}{5-1}} = \sqrt{\frac{14-12.8}{4}} = .5477$$

The 5-step approach to hypothesis testing

1. $H_0: \mu_d \geq 0$ and $H_1: \mu_d < 0$
2. $\alpha = .01$
3. The test statistic is $\bar{d}$.
4. $df = n - 1 = 5 - 1 = 4$ and α of $.01 \rightarrow t = -3.747$
5. Apply the decision rule.

$$t = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}} = \frac{-1.6}{\frac{.5477}{\sqrt{5}}} = \frac{-1.6}{.245} = -6.53$$

Reject H_0 because - 6.53 is beyond -3.747.

Training increased efficiency.

Note: H_0 points to the left and t is negative. Why? When scores increase, their difference is negative.